The number of partitions of an even number into sums of pairs of prime numbers.

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Abstract

The purpose of this work is to obtain exact and approximate formulas that calculate the number of partitions of even numbers into sums of pairs of prime numbers.

1. Introduction

Let's represent the sum of two prime numbers as an even number:

$$p_k + p_t = 2n \tag{1}$$

where p_k and p_t are prime numbers ($p_1 = 2$, $p_2 = 3$, $p_3 = 5$, $p_4 = 7$,...),
k and t are indices of prime numbers,
$2n$ is an even number,
$k, t, n \in N$.

The set of sums of prime numbers is represented in the form of Table 1.

Table 1: Even numbers obtained by pairwise summation of prime numbers

p_k	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47
2	4	-	-	-	-	-	-	-	-	-	-	-	-	-	-
3	-	6	8	10	14	16	20	22	26	32	34	40	44	46	50
5	-	8	10	12	16	18	22	24	28	34	36	42	46	48	52
7	-	10	12	14	18	20	24	26	30	36	38	44	48	50	54
11	-	14	16	18	22	24	28	30	34	40	42	48	52	54	58
13	-	16	18	20	24	26	30	32	36	42	44	50	54	56	60
17	-	20	22	24	28	30	34	36	40	46	48	54	58	60	64
19	-	22	24	26	30	32	36	38	42	48	50	56	60	62	66
23	-	26	28	30	34	36	40	42	46	52	54	60	64	66	70
29	-	32	34	36	40	42	46	48	52	58	60	66	70	72	76
31	-	34	36	38	42	44	48	50	54	60	62	68	72	76	78
37	-	40	42	44	48	50	54	56	60	66	68	74	78	80	84
41	-	44	46	48	52	54	58	60	64	70	72	78	82	84	88
43	-	46	48	50	54	56	60	62	66	72	74	80	84	86	90
47	-	50	52	54	58	60	64	66	70	76	78	84	88	90	94

Let $2n$ be an even number.

Let $PP(2n)$ be the exact number of partitions of an even number into sums of pairs of prime numbers.

Let $NSPP(2n)$ be the number of unique sums of pairs of prime numbers.

Let the number of prime numbers be l.

The number of unique sums of pairs of prime numbers forming even numbers will be no more than the following value:

$$NSPP(2n) \ < \ 1 \ + \ (1+2+3+...+l-1) \ = \ 1 \ + \ \frac{(l-1)\cdot l}{2} \ = \ 1 \ + \ \frac{l^2-l}{2} \ = \ \frac{l^2-l+2}{2} \tag{2}$$

Since l is the number of prime numbers in the range $[1;\ 2n]$, $l = \pi(2n)$.

Then replacing l by $\pi(2n)$ in formula (2), we obtain

$$NSPP(2n) \ \sim \ \frac{\big(\pi(2n)\big)^2 - \pi(2n) + 2}{2} \ = \ \frac{1}{2}\cdot\big(\pi(2n)\big)^2 \ - \ \frac{1}{2}\cdot\pi(2n) \ + \ 1 \tag{3}$$

Let us use the well-known Gauss formula to estimate the number of prime numbers:

$$\pi(2n) \ \sim \ \frac{2n}{log(2n)} \tag{4}$$

Let us substitute formula (4) into formula (3) and obtain the following expression:

$$NSPP(2n) \ \sim \ \frac{1}{2}\cdot\frac{4n^2}{\big(log(2n)\big)^2} \ - \ \frac{1}{2}\cdot\frac{2n}{log(2n)} \ + \ 1 \tag{5}$$

Let's find the estimate of the differential of the function $\Delta NSPP(2n)$.

Let $n \gg \Delta n = 1$, then

$$\Delta NSPP(2n) \ \sim \ NHPP(2n)'\cdot\Delta n \ = \ NSPP(2n)'\cdot 1 \ = \ NSPP(2n)' \tag{6}$$

$$NSPP(2n)' \ \sim \ \left(\frac{1}{2}\cdot\frac{4n^2}{\big(log(2n)\big)^2} \ - \ \frac{1}{2}\cdot\frac{2n}{log(2n)} \ + \ 1\right)' \ = \ \frac{\big(4n - log(2n)\big)\cdot\big(log(2n)-1\big)}{\big(log(2n)\big)^3} \tag{7}$$

For a rough estimate, the following formula can be used:

$$\Delta NSPP(2n) \ \sim \ NSPP(2n)' \ \sim \ \left\lceil \frac{\big(4n - log(2n)\big)\cdot\big(log(2n)-1\big)}{\big(log(2n)\big)^3} \right\rceil \tag{8}$$

The results of calculations show that at $2n \to \infty$ the following regularity is observed:

$$PP(2n) \;\sim\; \Delta NSPP(2n) \tag{9}$$

More precisely at $2n \to \infty$ the regularity will look as follows:

$$PP(2n) \;\sim\; \frac{1}{2} \cdot \Delta NSPP(2n) \tag{10}$$

Let's write it in more detailed form:

$$PP(2n) \;\sim\; \frac{1}{2} \cdot \left\lceil \frac{\big(4n - log(2n)\big) \cdot \big(log(2n) - 1\big)}{\big(log(2n)\big)^3} \right\rceil \;\sim\; \left\lceil \frac{\big(2n - 0.5 \cdot log(2n)\big) \cdot \big(log(2n) - 1\big)}{\big(log(2n)\big)^3} \right\rceil \tag{11}$$

The exact value of the partition function $PP(2n)$ can be written as follows:

$$PP(2n) \;=\; \left\lceil \mu \cdot \frac{\big(2n - 0.5 \cdot log(2n)\big) \cdot \big(log(2n) - 1\big)}{\big(log(2n)\big)^3} \right\rceil \tag{12}$$

In formula (12) μ is some coefficient.

If $2n \to \infty$, then formula (11) can be simplified:

$$PP(2n) \;\sim\; \left\lceil \frac{2n \cdot \big(log(2n) - 1\big)}{\big(log(2n)\big)^3} \right\rceil \tag{13}$$

Accordingly, formula (12) will be simplified to the following form:

$$PP(2n) \;=\; \left\lceil \nu \cdot \frac{2n \cdot \big(log(2n) - 1\big)}{\big(log(2n)\big)^3} \right\rceil \tag{14}$$

In formula (14) ν is some coefficient.

Now let us present the exact values of the $PP(2n)$ function, as well as the approximate estimates of the $PP(2n)$ function by formulas (11) and (13) and record them in Table 2.

Table 2: Number of partitions of even numbers into pairs of prime numbers

$2n$	$PP(2n)$	$\lceil\frac{(2n-0.5log(2n))(log(2n)-1)}{(log(2n))^3}\rceil$	$\lceil\frac{2n(log(2n)-1)}{(log(2n))^3}\rceil$
2	0	-1	-1
4	1	1	1
6	1	1	1
8	1	1	1
10	2	1	2
12	1	2	2
14	2	2	2
16	2	2	2
18	2	2	2
20	2	2	2
22	3	2	2
24	3	2	2
26	3	2	2
28	2	2	2
30	3	2	2
32	2	2	2
34	4	2	2
36	4	2	3
38	2	2	3
40	3	3	3
42	4	3	3
44	3	3	3
46	4	3	3
48	5	3	3
50	4	3	3
100	6	4	4
500	13	11	11
1000	28	18	18
5000	76	61	61
10000	127	106	106
50000	450	388	388
100000	810	689	689
500000	3052	2683	2683
1000000	5402	4860	4860
1500000	15164	6896	6896
2000000	9720	8847	8847
2500000	11701	10738	10738
5000000	21290	19653	19653
10000000	38807	36105	36105
50000000	158467	150127	150127
100000000	291400	278708	278708
500000000	1219610	1184026	1184026
1000000000	2274205	2216176	2216176
5000000000	9703556	9576174	9576174
10000000000	18200488	18042040	18042040
50000000000	79004202	79042004	79042004

2. Cumulative function of counting the number of hits of prime numbers in prime numbers.

Let an arbitrary even number $2n > 2$ or $2n \geq 4$ be given, where n is a natural number and $n \geq 2$.

Let $\pi(x)$ be the exact number of prime numbers in the interval $[0; x]$.

Let's try to solve our problem geometrically.

Consider a square with side length equal to $2n$.

Draw a line from the lower left vertex to the upper right vertex of the square.

The resulting slanted line is the segment dividing the square into two equal triangles.

Let's construct and analyze the diagram of decomposition of an even number into prime numbers.

The scheme of partitioning an even number into pairs of prime numbers (Figure 1) looks non-compactly.

Let us give a graphical representation of the decomposition of an even number into pairs of prime numbers in a more compact form.

Rotate the slanted line so that it becomes horizontal.

The result is two, symmetrical with respect to the central midpoint, upper and lower scales.

The length of the overlap of the two scales will be an even number.

The sum of the two numbers on the same vertical line of the lower and upper scale will also equal the same even number.

The total number of steps in the interval $[0; 2n]$ will be $2n/2 = n$.

Let the lower scale be stationary (fix the lower scale).

Let us consider the process of formation of even numbers and accumulation of prime numbers in the interval $[0; 2n]$ when the upper scale is shifted by a step equal to 2.

Successively shift the upper scale relative to the lower scale to the right by 2.

Each time the upper scale is shifted relative to the lower scale by a step equal to 2, the odd numbers of the upper scale will fall into the odd numbers of the lower scale, and the even numbers of the upper scale will fall into the even numbers of the lower scale.

Since the prime number 2 belongs to a subset of even numbers, and odd prime numbers belong to a subset of odd numbers, 2 will fall in even numbers (including the number 2), and odd prime numbers will fall in odd prime numbers.

Let NHPP be the exact number of hits of prime numbers in prime numbers.

As the upper scale moves, the number of prime numbers hitting prime numbers increases.

Obviously, the $NHPP(2n)$ function is increasing and cumulative.

Each subsequent offset includes previous hits and new hits.

Then the exact number of partitions of even number $2n$ into sums of pairs of prime numbers will be equal to the difference of cumulative hits of prime numbers into prime numbers generated by the current and previous scale offsets:

$$PP(2n) = \lceil 0.5 \cdot \Delta NHPP(2n) \rceil = \lceil 0.5 \cdot \big(NHPP(2n) - NHPP(2n-2) \big) \rceil \qquad (15)$$

where $\lceil \ \rceil$ is the rounding up operator,

coefficient 0.5 takes into account the symmetry property of partitioning an even number into prime numbers.

Partitions of even numbers into pairs of prime numbers

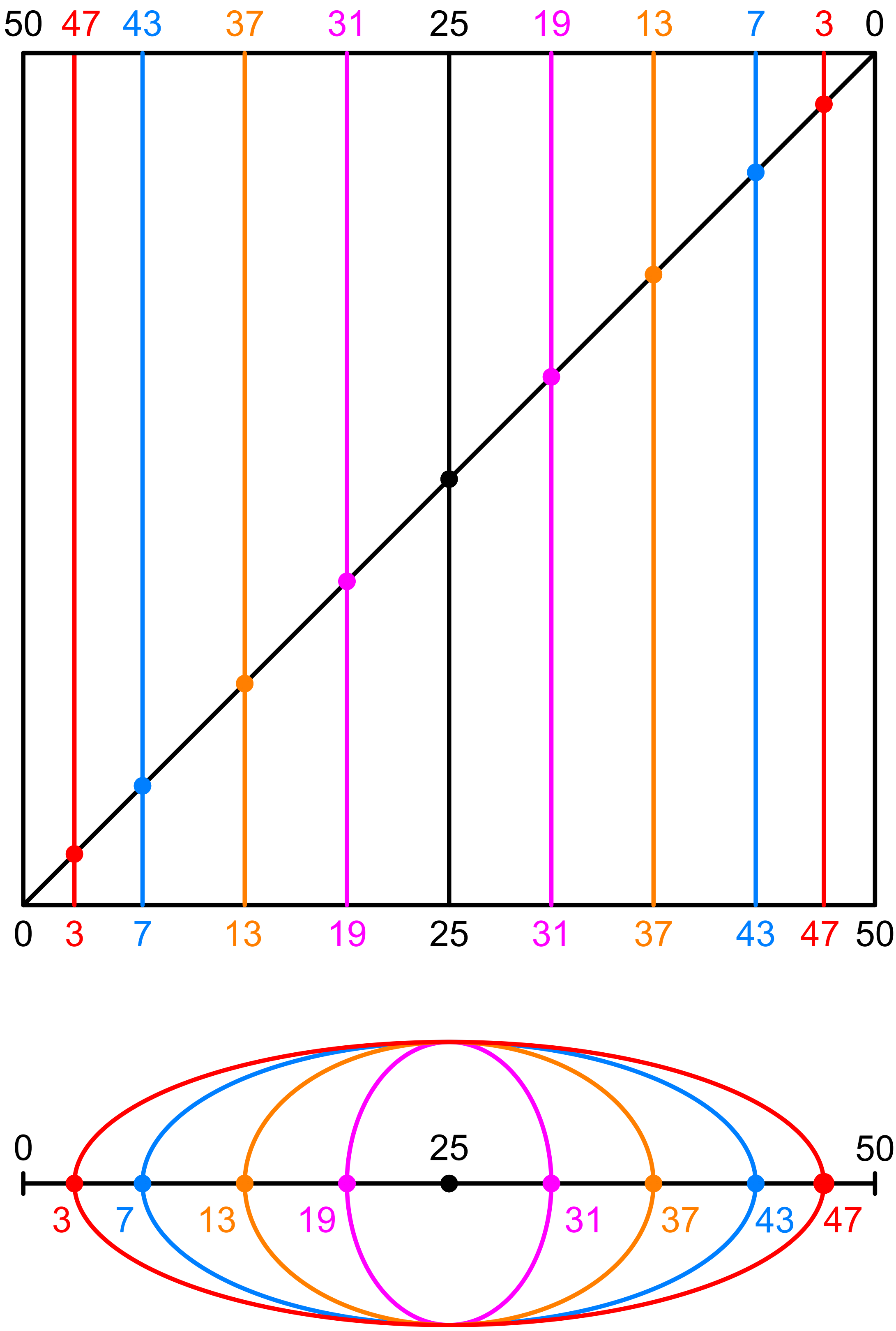

Figure 1. Partitioning the number 50 into pairs of prime numbers

Partitions of even numbers into pairs of prime numbers using shifted digital scales

Figure 2. Representing partitions of even numbers between 0 and 20

3.1. Derivation of the formula for the function $PP(2n)$.

Let's turn to the diagram (Figure 2) and consider the partitioning process in more detail.
The prime number 2 will go through all even numbers up to and including the number $(2n-2)$.
The prime number 3 will go through all even numbers up to and including the number $(2n-3)$.
The prime number 5 will go through all even numbers up to and including the number $(2n-5)$.
The prime number 7 will go through all even numbers up to and including the number $(2n-7)$.
....
The prime number p_k will go through all even numbers up to and including the number $(2n-p_k)$.

Since the set of odd prime numbers is a subset of odd numbers, the following conclusion can be drawn:
The prime number 2 of the upper scale will hit the prime number 2 of the lower scale once.
The prime number 3 of the upper scale will hit the prime numbers of the lower scale $\pi(2n-3)-1$ times.
The prime number 5 of the upper scale will hit the prime numbers of the lower scale $\pi(2n-5)-1$ times.
The prime number 7 of the upper scale will hit the prime numbers of the lower scale $\pi(2n-7)-1$ times.
....
The prime number p_k of the upper scale will hit the prime numbers of the lower scale $\pi(2n-p_k)-1$ times.

The index of the largest prime number will be $\pi(2n-3)$.
Accordingly, the largest prime number will be equal to $p_{\pi(2n-3)}$.
The sum of all hits of prime numbers of the upper scale in the prime numbers of the lower scale is equal:

$$1+\Big(\pi(2n-3)-1\Big)+\Big(\pi(2n-5)-1\Big)+\Big(\pi(2n-7)-1\Big)+...+\Big(\pi(2n-p_{\pi(2n-3)})-1\Big) \qquad (16)$$

or

$$1+\Big(\pi(2n-p_2)-1\Big)+\Big(\pi(2n-p_3)-1\Big)+\Big(\pi(2n-p_4)-1\Big)+...+\Big(\pi(2n-p_{\pi(2n-3)})-1\Big) \qquad (17)$$

Now we can write down the formula for the cumulative function of counting the number of hits of prime numbers in prime numbers:

$$NHPP(2n) \;=\; 1 \;+\; \sum_{k=2}^{\pi(2n-3)} \Big(\pi(2n-p_k)-1\Big) \qquad (18)$$

where p_k is a prime number ($p_1=2$, $p_2=3$, $p_3=5$, $p_4=7$,...),
k is the index of prime number,
$2n$ is a given even number,
$k, n \in N$.
The formula (18) is correct for $n \geq 3$ and $2n \geq 6$ since $\pi(2n-3) \geq 2$.
Let's simplify the above formula (18):

$$NHPP(2n) \;=\; 1 \;+\; \sum_{k=2}^{\pi(2n-3)} \pi(2n-p_k) \;-\; \sum_{k=2}^{\pi(2n-3)} 1 \qquad (19)$$

$$NHPP(2n) \;=\; 1 \;+\; \sum_{k=2}^{\pi(2n-3)} \pi(2n-p_k) \;-\; \Big(\pi(2n-3) \;-\; 1\Big) \tag{20}$$

$$NHPP(2n) \;=\; 1 \;+\; \sum_{k=2}^{\pi(2n-3)} \pi(2n-p_k) \;-\; \pi(2n-3) \;+\; 1 \tag{21}$$

$$NHPP(2n) \;=\; 2 \;-\; \pi(2n-3) \;+\; \sum_{k=2}^{\pi(2n-3)} \pi(2n-p_k) \tag{22}$$

The formula (22) is correct for $n \geq 3$ and $2n \geq 6$ since $\pi(2n-3) \geq 2$.

$$NHPP(2n-2) \;=\; 2 \;-\; \pi(2n-5) \;+\; \sum_{k=2}^{\pi(2n-5)} \pi(2n-2-p_k) \tag{23}$$

The formula (23) is correct for $n \geq 4$ and $2n \geq 8$ since $\pi(2n-5) \geq 2$.

Let's substitute the right parts of formulas (22) and (23) into the previously proposed formula (15) $PP(2n) \;=\; \lceil 0.5 \cdot \Delta NHPP(2n) \rceil \;=\; \lceil 0.5 \cdot \big(NHPP(2n) \;-\; NHPP(2n-2)\big) \rceil$ and we obtain the following expression:

$$\Delta NHPP(2n) \;=\; \sum_{k=2}^{\pi(2n-3)} \pi(2n-p_k) \;-\; \sum_{k=2}^{\pi(2n-5)} \pi(2n-2-p_k) \;-\; \pi(2n-3) \;+\; \pi(2n-5) \tag{24}$$

$$PP(2n) \;=\; \left\lceil 0.5 \cdot \left(\sum_{k=2}^{\pi(2n-3)} \pi(2n-p_k) \;-\; \sum_{k=2}^{\pi(2n-5)} \pi(2n-2-p_k) \;-\; \pi(2n-3) \;+\; \pi(2n-5) \right) \right\rceil \tag{25}$$

In the formula (25), $\lceil \; \rceil$ is the rounding up operator.
The formula (25) is correct for $n \geq 4$ and $2n \geq 8$ since $\pi(2n-5) \geq 2$.
Let's try to simplify formula (22).

$$NHPP(2n) \;=\; 2 \;-\; \pi(2n-3) \;+\; \pi(2n-p_2) \;+\; \sum_{k=3}^{\pi(2n-3)} \pi(2n-p_k) \tag{26}$$

$$NHPP(2n) \;=\; 2 \;-\; \pi(2n-3) \;+\; \pi(2n-3) \;+\; \sum_{k=3}^{\pi(2n-3)} \pi(2n-p_k) \tag{27}$$

$$NHPP(2n) \;=\; 2 \;+\; \sum_{k=3}^{\pi(2n-3)} \pi(2n-p_k) \tag{28}$$

The formula (28) is correct for $n \geq 4$ and $2n \geq 8$ since $\pi(2n-3) \geq 3$.

Now let's write down the formula for the number of hits for the previous even number:

$$NHPP(2n-2) \;=\; 2 \;+\; \sum_{k=3}^{\pi(2n-5)} \pi(2n-2-p_k) \tag{29}$$

The formula (29) is correct for $n \geq 5$ and $2n \geq 10$ since $\pi(2n-5) \geq 3$.

Let's substitute the right parts of formulas (28) and (29) into the above-mentioned formula (15):

$$PP(2n) \;=\; \left\lceil 0.5 \cdot \left(\left(2 \;+\; \sum_{k=3}^{\pi(2n-3)} \pi(2n-p_k) \right) \;-\; \left(2 \;+\; \sum_{k=3}^{\pi(2n-5)} \pi(2n-2-p_k) \right) \right) \right\rceil \tag{30}$$

And we get the formula for the exact number of partitions of an even number into pairs of prime numbers:

$$PP(2n) \;=\; \left\lceil 0.5 \cdot \left(\sum_{k=3}^{\pi(2n-3)} \pi(2n-p_k) \;-\; \sum_{k=3}^{\pi(2n-5)} \pi(2n-2-p_k) \right) \right\rceil \tag{31}$$

The formula (31) is correct for $n \geq 5$ and $2n \geq 10$ since $\pi(2n-5) \geq 3$.

Let's consider the following simple model:

$$\begin{cases} \sum\limits_{a}^{b-a} f(b-x) \;=\; f(b-a) + ... + f(a) \\ \\ \sum\limits_{a}^{b-a} f(x) \;=\; f(a) + ... + f(b-a) \\ \\ \sum\limits_{a}^{b-a} f(b-x) \;=\; \sum\limits_{a}^{b-a} f(x) \end{cases}$$

Let the following function be defined:

$$f(b-x) \;=\; \pi(2n-p_k) \tag{32}$$

and, accordingly:

$$f(x) \;\sim\; \pi(p_k) \;=\; k \tag{33}$$

Let's consider in formula (22) the sum of $\sum\limits_{k=2}^{\pi(2n-3)} \pi(2n-p_k)$.

$$\sum_{k=2}^{\pi(2n-3)} \pi(2n-p_k) \;=\; \pi(2n-p_2) + \pi(2n-p_3) + \pi(2n-p_4) + ... + \pi(2n-p_{\pi(2n-3)}) \tag{34}$$

For $2n \to \infty$ the following regularity is true:

$$p_{\pi(2n-3)} \;\to\; (2n-3) \tag{35}$$

That is, when $2n \to \infty$ we can assert the following:

$$\pi(2n - p_{\pi(2n-3)}) \;\to\; \pi(2n - (2n-3)) \;=\; \pi(3) \;=\; 2 \tag{36}$$

Then the estimation of the sum $\sum\limits_{k=2}^{\pi(2n-3)} \pi(2n - p_k)$ will have the following form:

$$\sum_{k=2}^{\pi(2n-3)} \pi(2n - p_k) \;\sim\; \pi(2n - p_2) + \pi(2n - p_3) + ... + 2 \tag{37}$$

For a rough estimate, let us represent the right-hand side of the expression in formula (37) as a sum of arithmetic progression with $\big(\pi(2n-3) - 1\big)$ elements:

$$\sum_{k=2}^{\pi(2n-3)} \pi(2n - p_k) \;\sim\; \pi(2n - p_2) + \pi(2n - p_3) + ... + 2 + 1 - 1 \tag{38}$$

$$\sum_{k=2}^{\pi(2n-3)} \pi(2n - p_k) \;\sim\; \sum_{k=2}^{\pi(2n-3)} \pi(p_k) \;=\; \sum_{k=2}^{\pi(2n-3)} k \;=\; -\,1 \;+\; \sum_{k=1}^{\pi(2n-3)} k \tag{39}$$

$$\sum_{k=2}^{\pi(2n-3)} \pi(2n - p_k) \;\sim\; -1 \;+\; \sum_{k=1}^{\pi(2n-3)} k \;=\; \frac{1}{2} \cdot \big(\pi(2n-3)\big) \cdot \big(\pi(2n-3) + 1\big) \;-\; 1 \tag{40}$$

$$\sum_{k=2}^{\pi(2n-3)} \pi(2n - p_k) \;\sim\; \frac{1}{2} \cdot \big(\pi(2n-3)\big)^2 \;+\; \frac{1}{2} * \pi(2n-3) \;-\; 1 \tag{41}$$

In formula (22) instead of the sum we substitute the expression obtained above:

$$NHPP(2n) \;\sim\; 2 \;-\; \pi(2n-3) \;+ \frac{1}{2} \cdot \big(\pi(2n-3)\big)^2 \;+\; \frac{1}{2} \cdot \pi(2n-3) \;-\; 1 \tag{42}$$

$$NHPP(2n) \;\sim\; \frac{1}{2} \cdot \big(\pi(2n-3)\big)^2 \;-\; \frac{1}{2} \cdot \pi(2n-3) \;+\; 1 \tag{43}$$

$$NHPP(2n) \;\sim\; \frac{1}{2} \cdot \pi(2n-3) \cdot \big(\pi(2n-3) \;-\; 1\big) \;+\; 1 \tag{44}$$

Let us use the well-known formula for estimating the number of prime numbers:

$$\pi(2n-3) \;\sim\; \frac{2n-3}{log(2n-3)} \tag{45}$$

Let us substitute formula (45) into formula (44) and obtain the following expression:

$$NHPP(2n) \;\sim\; \frac{1}{2} \cdot \frac{2n-3}{log(2n-3)} \cdot \left(\frac{2n-3}{log(2n-3)} \;-\; 1\right) \;+\; 1 \tag{46}$$

Let us find an estimate of the differential of the function $\Delta NHPP(2n)$.

Let $n \gg \Delta n = 1$, then

$$\Delta NHPP(2n) \;\sim\; NHPP(2n)' \cdot \Delta n \;=\; NHPP(2n)' \cdot 1 \;=\; NHPP(2n)' \tag{47}$$

$$NHPP(2n)' \;\sim\; \left(\frac{1}{2} \cdot \frac{2n-3}{log(2n-3)} \cdot \left(\frac{2n-3}{log(2n-3)} \;-\; 1\right) \;+\; 1\right)' \tag{48}$$

$$\Delta NHPP(2n) \;\sim\; NHPP(2n)' \;\sim\; \frac{\big(2 \cdot (2n-3) - log(2n-3)\big) \cdot \big(log(2n-3) - 1\big)}{\big(log(2n-3)\big)^3} \tag{49}$$

$$PP(2n) \;=\; \lceil 0.5 \cdot \Delta NHPP(2n) \rceil \;\sim\; \left\lceil 0.5 \cdot \frac{\big(2 \cdot (2n-3) - log(2n-3)\big) \cdot \big(log(2n-3) - 1\big)}{\big(log(2n-3)\big)^3} \right\rceil \tag{50}$$

An estimate of the number of partitions of an even number into pairs of prime numbers can be written as follows:

$$PP(2n) \;\sim\; \left\lceil \frac{\big(2n - 3 - 0.5 \cdot log(2n-3)\big) \cdot \big(log(2n-3) - 1\big)}{\big(log(2n-3)\big)^3} \right\rceil \tag{51}$$

The exact number of partitions of an even number into pairs of prime numbers can be written as follows:

$$PP(2n) \;\sim\; \left\lceil \mu \cdot \frac{\big(2n - 3 - 0.5 \cdot log(2n-3)\big) \cdot \big(log(2n-3) - 1\big)}{\big(log(2n-3)\big)^3} \right\rceil \tag{52}$$

In formula (52) μ is some coefficient.

Formula (51) is similar to formula (11) obtained earlier, and formula (52) is similar to formula (12).

Now let us calculate the exact values of the function $PP(2n)$ as well as the approximate estimates of the function $PP(2n)$ using formula (51) and record them in Table 3.

Table 3: Number of partitions of even numbers into pairs of prime numbers

$2n$	$PP(2n)$	$\lceil \frac{(2n-3-0.5log(2n-3))(log(2n-3)-1)}{(log(2n-3))^3} \rceil$
2	0	-
4	1	-
6	1	1
8	1	1
10	2	1
12	1	1
14	2	1
16	2	2
18	2	2
20	2	2
22	3	2
24	3	2
26	3	2
28	2	2
30	3	2
32	2	2
34	4	2
36	4	2
38	2	2
40	3	2
42	4	3
44	3	3
46	4	3
48	5	3
50	4	3
100	6	4
500	13	11
1000	28	18
5000	76	61
10000	127	106
50000	450	388
100000	810	689
500000	3052	2683
1000000	5402	4860
1500000	15164	6896
2000000	9720	8847
2500000	11701	10738
5000000	21290	19653
10000000	38807	36105
50000000	158467	150127
100000000	291400	278708
500000000	1219610	1184026
1000000000	2274205	2216176
5000000000	9703556	9576174
10000000000	18200488	18042040
50000000000	79004202	79042004

3.2. Derivation of the formula for the function $PP(2n)$.

Let us perform a small optimization of formula (22).

When the scales are shifted by an even number $2n$, each odd prime number $3, 5, ..., p_{\pi(n)}$ from the interval $(0; n]$ will fall into each such odd prime number $3, 5, ..., p_{\pi(n)}$. The number of these odd prime numbers is $\big(\pi(n) - 1\big)$. The prime number 2 will hit itself only 1 time.

Therefore, the first component of the formula for the number of hits of prime numbers in prime numbers at shifting numerical scales will look as follows:

$$1 + \Big(\pi(n) - 1\Big)^2 \tag{53}$$

Let's take advantage of the symmetric arrangement of numbers on the combined scales.

When the two scales are shifted by an even number $2n$, each odd prime number $3, 5, ..., p_{\pi(n)}$ will go right and left beyond the middle of the interval n only twice.

Therefore, the second component of the formula for the number of hits of prime numbers in prime numbers when the numerical scales are shifted will look as follows:

$$2 \cdot \sum_{k=2}^{\pi(n)} \Big(\big(\pi(2n - p_k) - 1\big) - \big(\pi(n) - 1\big)\Big) \tag{54}$$

Now add up formulas (53) and (54) and we get the following formula for the number of hits of prime numbers in prime numbers:

$$NHPP(2n) \;=\; 1 + \Big(\pi(n) - 1\Big)^2 + 2 \cdot \sum_{k=2}^{\pi(n)} \Big(\big(\pi(2n - p_k) - 1\big) - \big(\pi(n) - 1\big)\Big) \tag{55}$$

$$NHPP(2n) \;=\; 1 + \Big(\pi(n) - 1\Big)^2 + 2 \cdot \sum_{k=2}^{\pi(n)} \Big(\pi(2n - p_k) - \pi(n)\Big) \tag{56}$$

$$NHPP(2n) \;=\; 1 + \Big(\pi(n)\Big)^2 - 2\pi(n) + 1 + 2 \cdot \sum_{k=2}^{\pi(n)} \pi(2n - p_k) - 2 \cdot \sum_{k=2}^{\pi(n)} \pi(n) \tag{57}$$

$$NHPP(2n) \;=\; 2 + \Big(\pi(n)\Big)^2 - 2\pi(n) + 2 \cdot \sum_{k=2}^{\pi(n)} \pi(2n - p_k) - 2 \cdot \pi(n) \cdot \sum_{k=2}^{\pi(n)} 1 \tag{58}$$

$$NHPP(2n) \;=\; 2 + \Big(\pi(n)\Big)^2 - 2\pi(n) + 2 \cdot \sum_{k=2}^{\pi(n)} \pi(2n - p_k) - 2 \cdot \pi(n) \cdot \Big(\pi(n) - 1\Big) \tag{59}$$

$$NHPP(2n) \;=\; 2 + \Big(\pi(n)\Big)^2 - 2\pi(n) + 2 \cdot \sum_{k=2}^{\pi(n)} \pi(2n - p_k) - 2 \cdot \Big(\pi(n)\Big)^2 + 2\pi(n) \tag{60}$$

As a result, we will have the following expression:

$$NHPP(2n) \;=\; 2 - \Big(\pi(n)\Big)^2 + 2 \cdot \sum_{k=2}^{\pi(n)} \pi(2n - p_k) \tag{61}$$

The formula (61) is correct for $n \geq 3$ and $2n \geq 6$ since $\pi(n) \geq 2$.

$$NHPP(2n-2) \;=\; 2 \;-\; \Big(\pi(n-1)\Big)^2 \;+\; 2 \cdot \sum_{k=2}^{\pi(n-1)} \pi(2n-2-p_k) \tag{62}$$

The formula (62) is correct for $n \geq 4$ and $2n \geq 8$ since $\pi(n-1) \geq 2$.

Let us substitute the right parts of formulas (61) and (62) into the previously proposed formula (15) $PP(2n) = \lceil 0.5 \cdot \Delta NHPP(2n) \rceil = \lceil 0.5 \cdot \big(NHPP(2n) - NHPP(2n-2)\big) \rceil$ and we get this result:

$$\Delta NHPP(2n) \;= 2 \cdot \sum_{k=2}^{\pi(n)} \pi(2n-p_k) \;- 2 \cdot \sum_{k=2}^{\pi(n-1)} \pi(2n-2-p_k) \;-\; \Big(\pi(n)\Big)^2 \;+\; \Big(\pi(n-1)\Big)^2 \tag{63}$$

$$PP(2n) \;=\; \left\lceil 0.5 \cdot \left(2 \cdot \sum_{k=2}^{\pi(n)} \pi(2n-p_k) \;- 2 \cdot \sum_{k=2}^{\pi(n-1)} \pi(2n-2-p_k) \;-\; \Big(\pi(n)\Big)^2 \;+\; \Big(\pi(n-1)\Big)^2 \right) \right\rceil \tag{64}$$

$$PP(2n) \;=\; \left\lceil \sum_{k=2}^{\pi(n)} \pi(2n-p_k) \;-\; \sum_{k=2}^{\pi(n-1)} \pi(2n-2-p_k) \;-\; 0.5 \cdot \left(\Big(\pi(n)\Big)^2 \;-\; \Big(\pi(n-1)\Big)^2 \right) \right\rceil \tag{65}$$

In the formula (65), $\lceil\ \rceil$ is the rounding up operator.
The formula (65) is correct for $n \geq 4$ and $2n \geq 8$ since $\pi(n-1) \geq 2$.

Let us equate the right parts of formulas (22) and (61) and obtain such an identical equation:

$$2 \;-\; \pi(2n-3) \;+\; \sum_{k=2}^{\pi(2n-3)} \pi(2n-p_k) \;=\; 2 \;-\; \Big(\pi(n)\Big)^2 \;+\; 2 \cdot \sum_{k=2}^{\pi(n)} \pi(2n-p_k) \tag{66}$$

$$-\pi(2n-3) \;+\; \sum_{k=2}^{\pi(2n-3)} \pi(2n-p_k) \;=\; -\Big(\pi(n)\Big)^2 \;+\; 2 \cdot \sum_{k=2}^{\pi(n)} \pi(2n-p_k) \tag{67}$$

$$\sum_{k=2}^{\pi(2n-3)} \pi(2n-p_k) \;-\; 2 \cdot \sum_{k=2}^{\pi(n)} \pi(2n-p_k) \;=\; \pi(2n-3) \;-\; \Big(\pi(n)\Big)^2 \tag{68}$$

or

$$2 \cdot \sum_{k=2}^{\pi(n)} \pi(2n-p_k) \;-\; \sum_{k=2}^{\pi(2n-3)} \pi(2n-p_k) \;=\; \Big(\pi(n)\Big)^2 \;-\; \pi(2n-3) \tag{69}$$

The formula (69) is correct for $n \geq 3$ and $2n \geq 6$ since $\pi(n) \geq 2$ and $\pi(2n-3) \geq 2$.

Now we equate the right-hand sides of formulas (28) and (61) and obtain a more interesting identity equation:

$$2 \;+\; \sum_{k=3}^{\pi(2n-3)} \pi(2n-p_k) \;=\; 2 \;-\; \Big(\pi(n)\Big)^2 \;+\; 2 \cdot \sum_{k=2}^{\pi(n)} \pi(2n-p_k) \tag{70}$$

$$\sum_{k=3}^{\pi(2n-3)} \pi(2n-p_k) \;=\; -\Big(\pi(n)\Big)^2 \;+\; 2\cdot\sum_{k=2}^{\pi(n)} \pi(2n-p_k) \tag{71}$$

$$2\cdot\sum_{k=2}^{\pi(n)} \pi(2n-p_k) \;-\; \sum_{k=3}^{\pi(2n-3)} \pi(2n-p_k) \;=\; \big(\pi(n)\big)^2 \tag{72}$$

The equation (72) is correct for $n \geq 4$ and $2n \geq 8$ since $\pi(2n-3) \geq 3$.

Let's consider the sum component in the right part of formula (61):

$$\sum_{k=2}^{\pi(n)} \pi(2n-p_k) \;=\; \pi(2n-p_2) \;+\; \pi(2n-p_3) \;+ .... +\; \pi(2n-p_{\pi(n)}) \tag{73}$$

Or the same thing:

$$\sum_{k=2}^{\pi(n)} \pi(2n-p_k) \;=\; \pi(2n-3) \;+\; \pi(2n-5) \;+ .... +\; \pi(2n-p_{\pi(n)}) \tag{74}$$

At $n \to \infty$ the formula (74) will have asymptotic character.
That is, the following phenomenon can be observed:

$$p_{\pi(n)} \;\to\; n \tag{75}$$

or

$$p_{\pi(n)} \;\sim\; n \tag{76}$$

$$\pi(2n-p_{\pi(n)}) \;\to\; \pi(2n-n) \;=\; \pi(n) \tag{77}$$

or

$$\pi(2n-p_{\pi(n)}) \;\sim\; \pi(2n-n) \;=\; \pi(n) \tag{78}$$

Taking into account the above assumptions, we simplify the sum component in the right-hand side of formula (74):

$$\pi(2n-3) + ... + \pi(2n-p_{\pi(n)}) \;\sim\; \pi(2n-3) + ... + \pi(n) \tag{79}$$

For a rough estimate, let us represent the right-hand side of the expression in formula (79) as the sum of an arithmetic progression with $\big(\pi(n)-1\big)$ elements:

$$\pi(2n-3) + ... + \pi(n) \;\sim\; \frac{\pi(2n-3)+\pi(n)}{2} * \big(\pi(n)-1\big) \tag{80}$$

Let's substitute the result of formula (80) into the right part of formula (74):

$$\sum_{k=2}^{\pi(n)} \pi(2n-p_k) \ \sim \ \frac{\pi(2n-3)+\pi(n)}{2} \cdot \big(\pi(n)-1\big) \tag{81}$$

accordingly:

$$2 \cdot \sum_{k=2}^{\pi(n)} \pi(2n-p_k) \ \sim \ \big(\pi(2n-3)+\pi(n)\big) \cdot \big(\pi(n)-1\big) \tag{82}$$

Let us expand the expression in parentheses for the right part of formula (82):

$$\big(\pi(2n-3)+\pi(n)\big) \cdot \big(\pi(n)-1\big) \ = \ \pi(2n-3)\cdot\pi(n) \ - \ \pi(2n-3) \ + \ \big(\pi(n)\big)^2 \ - \ \pi(n) \tag{83}$$

As a result, we get an expression for the sum component:

$$2 \cdot \sum_{k=2}^{\pi(n)} \pi(2n-p_k) \ \sim \ \pi(2n-3)\cdot\pi(n) \ - \ \pi(2n-3) \ + \ \big(\pi(n)\big)^2 \ - \ \pi(n) \tag{84}$$

Now let us replace the sum component in the right-hand side of formula (61) with the right-hand side of formula (84):

$$NHPP(2n) \ \sim \ 2 \ - \ \Big(\pi(n)\Big)^2 \ + \ \pi(2n-3) * \pi(n) \ - \ \pi(2n-3) \ + \ \Big(\pi(n)\Big)^2 \ - \ \pi(n) \tag{85}$$

As a result, we get an expression for the sum component:

$$NHPP(2n) \ \sim \ 2 \ + \ \pi(2n-3)\cdot\pi(n) \ - \ \pi(2n-3) \ - \ \pi(n) \tag{86}$$

To simplify formula (86), we take the common multipliers out of brackets and obtain the following expression:

$$NHPP(2n) \ \sim \ \big(\pi(2n-3)-1\big) \cdot \big(\pi(n)-1\big) \ + \ 1 \tag{87}$$

Let's write down the formula for the number of hits of prime numbers in prime numbers at shifting of numerical scales will be in the following form:

$$NHPP(2n) \ = \ \big\lceil \mu \cdot \big(\pi(2n-3)-1\big) \cdot \big(\pi(n)-1\big) \big\rceil \ + \ 1 \tag{88}$$

Let's use a well-known formula for estimating the number of prime numbers:

$$\pi(2n-3) \ \sim \ \frac{2n-3}{log(2n-3)} \tag{89}$$

Let us substitute formula (89) into formula (88) and obtain this expression:

$$NHPP(2n) \;=\; \left\lceil \mu \cdot \left(\frac{2n-3}{log(2n-3)} \;-\; 1 \right) \cdot \left(\frac{n}{log(n)} \;-\; 1 \right) \right\rceil \;+\; 1 \tag{90}$$

We can write down a formula for approximating the function $NHPP(2n)$:

$$NHPP(2n) \;\sim\; \left\lceil \left(\frac{2n-3}{log(2n-3)} \;-\; 1 \right) \cdot \left(\frac{n}{log(n)} \;-\; 1 \right) \right\rceil \;+\; 1 \tag{91}$$

Let us find an estimate of the differential of the function $\Delta NHPP(2n)$:
Let $n \gg \Delta n = 1$, then

$$\Delta NHPP(2n) \;\sim\; NHPP(2n)' \cdot \Delta n \;=\; NHPP(2n)' \cdot 1 \;=\; NHPP(2n)' \tag{92}$$

$$NHPP(2n)' \;\sim\; \left\lceil \frac{2 \cdot \big(log(2n-3) - 1\big)}{\big(log(2n-3)\big)^2} \cdot \left(\frac{n}{log(n)} - 1 \right) \;+\; \left(\frac{2n-3}{log(2n-3)} - 1 \right) \cdot \frac{log(n) - 1}{\big(log(n)\big)^2} \right\rceil \tag{93}$$

For values $n \gg 1$, formula (93) can be simplified:

$$\Delta NHPP(2n) \;\sim\; NHPP(2n)' \;\sim\; \left\lceil \frac{4n \;-\; log(2 \cdot n^3)}{(log(2n)) \cdot log(n)} \right\rceil \tag{94}$$

The formula for the number of partitions of an even number into pairs of prime numbers would look like this:

$$PP(2n) \;=\; \lceil 0.5 \cdot \Delta NHPP(2n) \rceil \;\sim\; \left\lceil 0.5 \cdot \frac{4n \;-\; log(2 \cdot n^3)}{(log(2n)) \cdot log(n)} \right\rceil \tag{95}$$

$$PP(2n) \;\sim\; \left\lceil \frac{2n \;-\; 0.5 \cdot log(2 \cdot n^3)}{(log(2n)) \cdot log(n)} \right\rceil \tag{96}$$

$$PP(2n) \;=\; \left\lceil \mu \cdot \frac{2n \;-\; 0.5 \cdot log(2 \cdot n^3)}{(log(2n)) \cdot log(n)} \right\rceil \tag{97}$$

In the formula (97), μ is some coefficient.

Let us now calculate the exact values of the $PP(2n)$ function as well as the approximate estimates of the $PP(2n)$ function using formula (96) and record them in Table 4.

Table 4: Number of partitions of even numbers into pairs of prime numbers.

$2n$	$PP(2n)$	$\lceil \frac{2n-0.5log(2\cdot n^3)}{(log(2n))\cdot log(n)} \rceil$
2	0	-
4	1	3
6	1	3
8	1	3
10	2	3
12	1	3
14	2	3
16	2	3
18	2	3
20	2	3
22	3	3
24	3	3
26	3	3
28	2	3
30	3	3
32	2	3
34	4	3
36	4	4
38	2	4
40	3	4
42	4	4
44	3	4
46	4	4
48	5	4
50	4	4
100	6	6
500	13	15
1000	28	24
5000	76	75
10000	127	128
50000	450	457
100000	810	803
500000	3052	3066
1000000	5402	5516
1500000	15164	7797
2000000	9720	9978
2500000	11701	12088
5000000	21290	22004
10000000	38807	40222
50000000	158467	165576
100000000	291400	306229
500000000	1219610	1290917
1000000000	2274205	2409120
5000000000	9703556	10346188
10000000000	18200488	19446570
50000000000	79004202	84771399

4. Calculating values of the function of counting partitions of even numbers into sums of pairs of prime numbers.

Let us calculate with the help of formulas (61), (62), (65) several exact values of the function $PP(2n)$:

$PP(2) = 0$
$PP(4) = \lceil 0.5 \cdot (1 - 0) \rceil = \lceil 0.5 \rceil = 1$
$PP(6) = \lceil 0.5 \cdot (2 - 1) \rceil = \lceil 0.5 \rceil = 1$
$PP(8) = \lceil 0.5 \cdot (4 - 2) \rceil = \lceil 0.5 \cdot 2 \rceil = \lceil 1 \rceil = 1$
$PP(10) = \lceil 0.5 \cdot (7 - 4) \rceil = \lceil 0.5 \cdot 3 \rceil = \lceil 1.5 \rceil = 2$
$PP(12) = \lceil 0.5 \cdot (9 - 7) \rceil = \lceil 0.5 \cdot 2 \rceil = \lceil 1 \rceil = 1$
$PP(14) = \lceil 0.5 \cdot (12 - 9) \rceil = \lceil 0.5 \cdot 3 \rceil = \lceil 1.5 \rceil = 2$
$PP(16) = \lceil 0.5 \cdot (16 - 12) \rceil = \lceil 0.5 \cdot 4 \rceil = \lceil 2 \rceil = 2$
$PP(18) = \lceil 0.5 \cdot (20 - 16) \rceil = \lceil 0.5 \cdot 4 \rceil = \lceil 2 \rceil = 2$
$PP(20) = \lceil 0.5 \cdot (24 - 20) \rceil = \lceil 0.5 \cdot 4 \rceil = \lceil 2 \rceil = 2$
$PP(22) = \lceil 0.5 \cdot (29 - 24) \rceil = \lceil 0.5 \cdot 5 \rceil = \lceil 2.5 \rceil = 3$
$PP(24) = \lceil 0.5 \cdot (35 - 29) \rceil = \lceil 0.5 \cdot 6 \rceil = \lceil 3 \rceil = 3$
$PP(26) = \lceil 0.5 \cdot (40 - 35) \rceil = \lceil 0.5 \cdot 5 \rceil = \lceil 2.5 \rceil = 3$
$PP(28) = \lceil 0.5 \cdot (44 - 40) \rceil = \lceil 0.5 \cdot 4 \rceil = \lceil 2 \rceil = 2$
$PP(30) = \lceil 0.5 \cdot (50 - 44) \rceil = \lceil 0.5 \cdot 6 \rceil = \lceil 3 \rceil = 3$
$PP(32) = \lceil 0.5 \cdot (54 - 50) \rceil = \lceil 0.5 \cdot 4 \rceil = \lceil 2 \rceil = 2$
$PP(34) = \lceil 0.5 \cdot (61 - 54) \rceil = \lceil 0.5 \cdot 7 \rceil = \lceil 3.5 \rceil = 4$
$PP(36) = \lceil 0.5 \cdot (69 - 61) \rceil = \lceil 0.5 \cdot 8 \rceil = \lceil 4 \rceil = 4$
$PP(38) = \lceil 0.5 \cdot (72 - 69) \rceil = \lceil 0.5 \cdot 3 \rceil = \lceil 1.5 \rceil = 2$
$PP(40) = \lceil 0.5 \cdot (78 - 72) \rceil = \lceil 0.5 \cdot 6 \rceil = \lceil 3 \rceil = 3$
$PP(42) = \lceil 0.5 \cdot (86 - 78) \rceil = \lceil 0.5 \cdot 8 \rceil = \lceil 4 \rceil = 4$
$PP(44) = \lceil 0.5 \cdot (92 - 86) \rceil = \lceil 0.5 \cdot 6 \rceil = \lceil 3 \rceil = 3$
$PP(46) = \lceil 0.5 \cdot (99 - 92) \rceil = \lceil 0.5 \cdot 7 \rceil = \lceil 3.5 \rceil = 4$
$PP(48) = \lceil 0.5 \cdot (109 - 99) \rceil = \lceil 0.5 \cdot 10 \rceil = \lceil 5 \rceil = 5$
$PP(50) = \lceil 0.5 \cdot (117 - 109) \rceil = \lceil 0.5 \cdot 8 \rceil = \lceil 4 \rceil = 4$
$PP(100) = \lceil 0.5 \cdot (361 - 349) \rceil = \lceil 0.5 \cdot 12 \rceil = \lceil 6 \rceil = 6$
$PP(500) = \lceil 0.5 \cdot (5139 - 5113) \rceil = \lceil 0.5 \cdot 26 \rceil = \lceil 13 \rceil = 13$
$PP(1000) = \lceil 0.5 \cdot (16349 - 16293) \rceil = \lceil 0.5 \cdot 56 \rceil = \lceil 28 \rceil = 28$
$PP(5000) = \lceil 0.5 \cdot (254879 - 254727) \rceil = \lceil 0.5 \cdot 152 \rceil = \lceil 76 \rceil = 76$
$PP(10000) = \lceil 0.5 \cdot (850833 - 850579) \rceil = \lceil 0.5 \cdot 254 \rceil = \lceil 127 \rceil = 127$
$PP(50000) = \lceil 0.5 \cdot (14634584 - 14633684) \rceil = \lceil 0.5 \cdot 900 \rceil = \lceil 450 \rceil = 450$
$PP(100000) = \lceil 0.5 \cdot (50728833 - 50727213) \rceil = \lceil 0.5 \cdot 1620 \rceil = \lceil 810 \rceil = 810$
$PP(500000) = \lceil 0.5 \cdot (939509218 - 939503114) \rceil = \lceil 0.5 \cdot 6104 \rceil = \lceil 3052 \rceil = 3052$
$PP(1000000) = \lceil 0.5 \cdot (3343718028 - 3343707224) \rceil = \lceil 0.5 \cdot 10804 \rceil = \lceil 5402 \rceil = 5402$
$PP(1500000) = \lceil 0.5 \cdot (7046952560 - 7046922232) \rceil = \lceil 0.5 \cdot 30328 \rceil = \lceil 15164 \rceil = 15164$
$PP(2000000) = \lceil 0.5 \cdot (11974338072 - 11974318632) \rceil = \lceil 0.5 \cdot 19440 \rceil = \lceil 9720 \rceil = 9720$
$PP(2500000) = \lceil 0.5 \cdot (18078516351 - 18078492949) \rceil = \lceil 0.5 \cdot 23402 \rceil = \lceil 11701 \rceil = 11701$

For more clarity, let's record the results in Table 5.

Table 5: Counting the number of partitions of even numbers into pairs of prime numbers

$2n$	$NHPP(2n)$	$NHPP(2n-2)$	$PP(2n)$
2	0	0	0
4	1	0	1
6	2	1	1
8	4	2	1
10	7	4	2
12	9	7	1
14	12	9	2
16	16	12	2
18	20	16	2
20	24	20	2
22	29	24	3
24	35	29	3
26	40	35	3
28	44	40	2
30	50	44	3
32	54	50	2
34	61	54	4
36	69	61	4
38	72	69	2
40	78	72	3
42	86	78	4
44	92	86	3
46	99	92	4
48	109	99	5
50	117	109	4
100	361	349	6
500	5139	5113	13
1000	16349	16293	28
5000	254879	254727	76
10000	850833	850579	127
50000	14634584	14633684	450
100000	50728833	50727213	810
500000	939509218	939503114	3052
1000000	3343718028	3343707224	5402
1500000	7046952560	7046922232	15164
2000000	11974338072	11974318632	9720
2500000	18078516351	18078492949	11701

5. Remarks on Goldbach's problem.

Suppose that all even numbers greater than 2, that is, the numbers 4, 6, 8, 10, 12, ... can be represented as the sum of two prime numbers.

In order for every even number ≥ 4 to be represented as the sum of two prime numbers, it is necessary for the prime numbers to hit the prime numbers at each step of the scale offset (Figure 2).

For any even number ≥ 4 to be representable as a sum of two prime numbers, it is necessary for the partition function $PP(2n)$ to be greater than zero.

That is, the condition of fulfillment of the binary Goldbach conjecture can be written as follows:

$$PP(2n) \ > \ 0 \tag{98}$$

Using formula (25), let us write the above expression in a more detailed form:

$$PP(2n) \ = \ \left\lceil 0.5 \cdot \left(\sum_{k=2}^{\pi(2n-3)} \pi(2n-p_k) \ - \sum_{k=2}^{\pi(2n-5)} \pi(2n-2-p_k) \ - \ \pi(2n-3) \ + \ \pi(2n-5) \right) \right\rceil \ > \ 0 \tag{99}$$

In the formula (99), $\lceil \ \rceil$ is the rounding up operator.

Formula (99) will be true if the following inequality is true:

$$\sum_{k=2}^{\pi(2n-3)} \pi(2n-p_k) \ - \sum_{k=2}^{\pi(2n-5)} \pi(2n-2-p_k) \ - \ \pi(2n-3) \ + \ \pi(2n-5) \ > \ 0 \tag{100}$$

$$\sum_{k=2}^{\pi(2n-3)} \pi(2n-p_k) \ - \sum_{k=2}^{\pi(2n-5)} \pi(2n-2-p_k) \ > \ \pi(2n-3) \ - \ \pi(2n-5) \tag{101}$$

$$\pi(2n-p_2) + \sum_{k=3}^{\pi(2n-3)} \pi(2n-p_k) - \pi(2n-2-p_2) - \sum_{k=3}^{\pi(2n-5)} \pi(2n-2-p_k) \ > \ \pi(2n-3) - \pi(2n-5) \tag{102}$$

$$\pi(2n-3) + \sum_{k=3}^{\pi(2n-3)} \pi(2n-p_k) - \pi(2n-2-3) - \sum_{k=3}^{\pi(2n-5)} \pi(2n-2-p_k) \ > \ \pi(2n-3) - \pi(2n-5) \tag{103}$$

$$\pi(2n-3) \ + \sum_{k=3}^{\pi(2n-3)} \pi(2n-p_k) \ - \ \pi(2n-5) \ - \sum_{k=3}^{\pi(2n-5)} \pi(2n-2-p_k) \ > \ \pi(2n-3) \ - \ \pi(2n-5) \tag{104}$$

$$\sum_{k=3}^{\pi(2n-3)} \pi(2n-p_k) \ - \sum_{k=3}^{\pi(2n-5)} \pi(2n-2-p_k) \ > \ 0 \tag{105}$$

Inequality (101) will be true if inequality (105) is true.

Since the function $\pi(x)$ is non-decreasing and has a step graph, the following inequality is true:

$$\pi(2n-3) \ \geq \ \pi(2n-5) \tag{106}$$

Given the above, inequality (105) will be true if the following inequality is true:

$$\sum_{k=3}^{\pi(2n-5)} \pi(2n-p_k) \ - \sum_{k=3}^{\pi(2n-5)} \pi(2n-2-p_k) \ > \ 0 \tag{107}$$

$$\sum_{k=3}^{\pi(2n-5)} \big(\pi(2n-p_k) \ - \ \pi(2n-2-p_k)\big) \ > \ 0 \tag{108}$$

$$\sum_{k=3}^{\pi(2n-5)} \big(\pi(2n-p_k) \ - \ \pi(2n-p_k-2)\big) \ > \ 0 \tag{109}$$

Suppose that the Goldbach conjecture is incorrect and inequality (109) is incorrect and the following equality is true:

$$\sum_{k=3}^{\pi(2n-5)} \big(\pi(2n-p_k) \ - \ \pi(2n-p_k-2)\big) \ = \ 0 \tag{110}$$

Equation (110) will be true if the following equation is true for all indices $k \in [3;\ \pi(2n-5)]$:

$$\pi(2n-p_k) \ - \ \pi(2n-p_k-2) \ = \ 0 \tag{111}$$

or

$$\pi(2n-p_k) \ = \ \pi(2n-p_k-2) \tag{112}$$

Let us simplify the form of writing equation (112). Let $(2n-p_k) = x_k$, then equation (112) will look like this:

$$\pi(x_k) \ = \ \pi(x_k-2) \tag{113}$$

Equation (113) will be true if x_k is an odd composite number.

A composite odd number is always between the two nearest odd prime numbers (for example: $7 < 9 < 11$).

In order for equation (113) to hold, it is necessary that all numbers $x_k = (2n-p_k)$ for the entire component of the sum fall only inside the prime intervals that are greater than 2 and do not fall on the boundaries of the prime intervals (Figure 3).

This is where we should stop.

The proof of Goldbach's conjecture will not be considered in this paper.

Now, for convenience, we introduce the following notations:

$$f(2n) = \sum_{k=3}^{\pi(2n-5)} \pi(2n-p_k) \tag{114}$$

$$f(2n-2) = \sum_{k=3}^{\pi(2n-5)} \pi(2n-p_k-2) \tag{115}$$

$$\Delta f(2n) = \sum_{k=3}^{\pi(2n-5)} \pi(2n-p_k) \ - \ \sum_{k=3}^{\pi(2n-5)} \pi(2n-2-p_k) \tag{116}$$

Now let us calculate the exact values of functions $f(2n)$, $f(2n-2)$, $\Delta f(2n))$ using formulas (114), (115), (116) and record them in Table 6.

Table 6: The function of hitting odd prime numbers in odd prime numbers

$2n$	$f(2n) = \sum_{k=3}^{\pi(2n-5)} \pi(2n - p_k)$	$f(2n-2) = \sum_{k=3}^{\pi(2n-5)} \pi(2n - p_k - 2)$	$\Delta f(2n) = f(2n) - f(2n-2)$
10	3	2	1
12	7	5	2
14	8	7	1
16	12	10	2
18	18	14	4
20	20	18	2
22	25	22	3
24	33	27	6
26	36	33	3
28	42	38	4
30	48	42	6
32	50	48	2
34	57	52	5
36	67	59	8
38	70	67	3
40	74	70	4
42	84	76	8
44	88	84	4
46	95	90	5
48	107	97	10
50	113	107	6
52	121	115	6
54	131	121	10
56	135	131	4
58	144	137	7
60	156	144	12
62	159	156	3
64	169	161	8
66	183	171	12
68	187	183	4
70	195	187	8
72	209	197	12
74	216	209	7
76	226	218	8
78	242	228	14
80	250	242	8
82	257	250	7
84	275	259	16
86	282	275	7
88	292	284	8
90	310	292	18
92	316	310	6
94	327	318	9
96	341	327	14
98	347	341	6
100	357	347	10

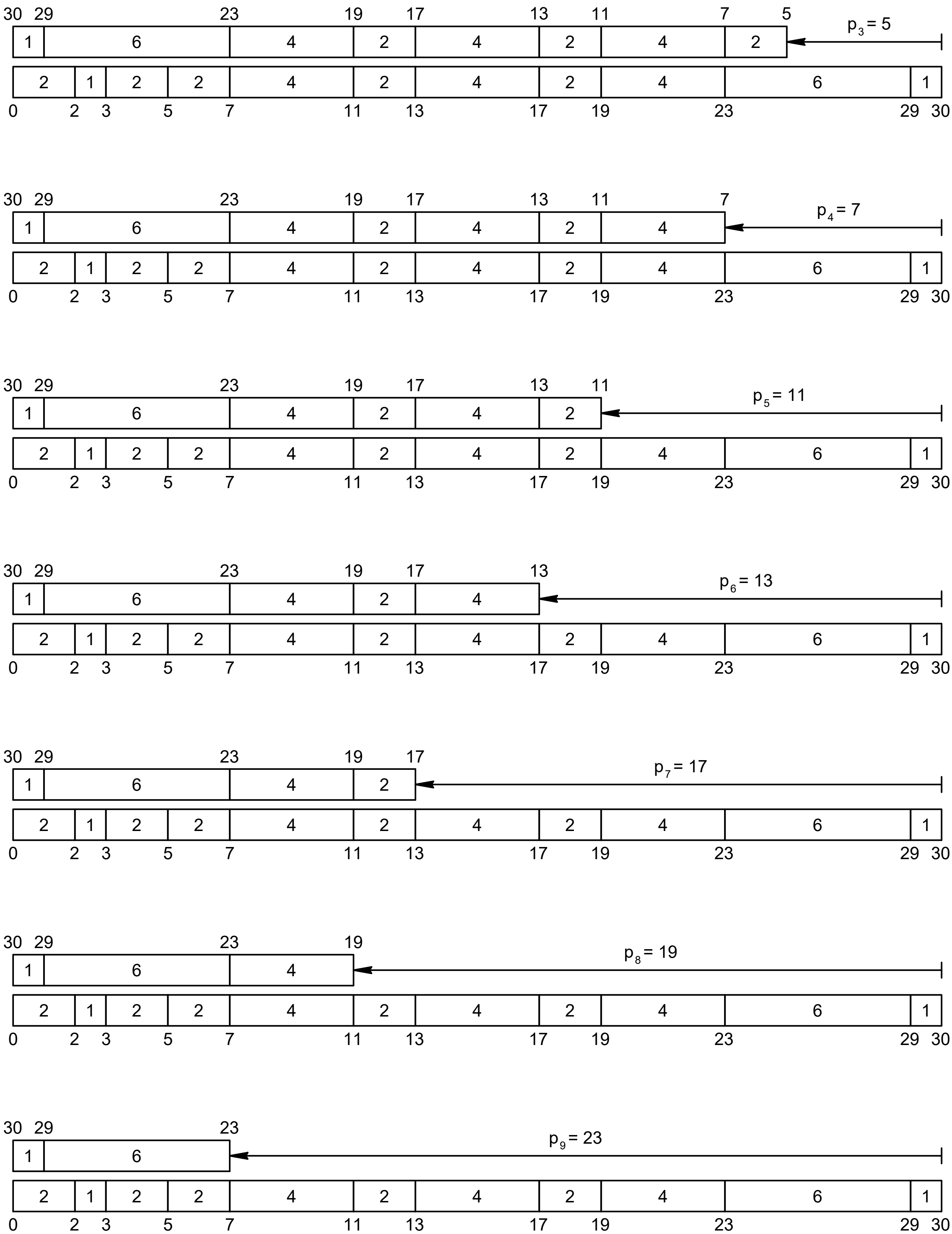

Figure 3. Scheme of hitting prime numbers p_k into prime gaps.

6. Other formulas for the partition function $PP(2n)$.

Suppose that an even number $2n$ can be decomposed into the sum of two prime numbers x and $(2n - x)$.

Let $TPN(x)$ be a function for testing the prime of a number.

If the number x is prime, then let $TPN(x) = 1$. If x is composite, then $TPN(x) = 0$.

If x and $(2n - x)$ are prime numbers, then the following conditions must be satisfied:

$$\begin{cases} TPN(x) & = & 1 \\ TPN(2n-x) & = & 1 \\ TPN(x) \cdot TPN(2n-x) & = & 1 \end{cases}$$

In view of the above, the partitioning function can be represented as follows:

$$PP(2n) \;=\; \sum_{k=2}^{n} TPN(k) \cdot TPN(2n-k) \tag{117}$$

If it is known in advance that all values of x are prime numbers p_k, then formula (117) can be simplified to the following form:

$$PP(2n) \;=\; \sum_{k=1}^{\pi(n)} TPN(2n-p_k) \tag{118}$$

If we consider only odd numbers $x = 2k + 1$, formula (117) takes the following form:

$$PP(2n) \;=\; \sum_{k=1}^{\lfloor (n-1)/2 \rfloor} TPN(2k+1) \cdot TPN(2n-2k-1) \tag{119}$$

The formula (119) is correct for $n \geq 3$ and $2n \geq 6$.

Suppose that an even number $2n$ can be decomposed into the sum of two prime numbers in the form $(n - k)$ и $(n + k)$.

Then both of these numbers are both prime numbers at the same time.

Here it is necessary to do a simplicity check on two numbers $(n - k)$ and $(n + k)$ simultaneously.

The partition function, taking into account the above, will have the following form:

$$PP(2n) \;=\; \sum_{k=0}^{n-2} TPN(n-k) \cdot TPN(n+k) \tag{120}$$

If n is an odd number, and the numbers $(n-2k)$ and $(n+2k)$ are also odd, then the modification of formula (120) will look as follows:

$$PP(2n) \;=\; \sum_{k=0}^{(n-3)/2} TPN(n-2k) \cdot TPN(n+2k) \tag{121}$$

The formula (121) is correct for $n \geq 3$ and $2n \geq 6$.

If n is an even number and the numbers $(n-2k-1)$ and $(n+2k+1)$ are odd, then the modification of formula (120) will look as follows:

$$PP(2n) \;=\; \sum_{k=0}^{(n-4)/2} TPN(n-2k-1) \cdot TPN(n+2k+1) \tag{122}$$

The formula (122) is correct for $n \geq 4$ and $2n \geq 8$.

Let us construct a more detailed formula for calculating the number of partitions of an even number into pairs of prime numbers using formula (117).

If a prime number k is given, the following formulas are valid:

$$\pi(p_k) = k \tag{123}$$

$$p_{\pi(p_k)} = p_k \tag{124}$$

$$\frac{p_{\pi(p_k)}}{p_k} = 1 \tag{125}$$

If in formula (125) we substitute p_k instead of k (assuming that k is a prime number), we obtain the following expression:

$$\frac{p_{\pi(k)}}{k} = 1 \tag{126}$$

Now in formula (126) we substitute k instead of $2n-k$ (assuming that $2n-k$ is a prime number) and obtain the following expression:

$$\frac{p_{\pi(2n-k)}}{2n-k} = 1 \tag{127}$$

The functions for checking the numbers k and $(2n-k)$ for simplicity will look like this:

$$TPN(k) = \left\lfloor \frac{p_{\pi(k)}}{k} \right\rfloor \tag{128}$$

$$TPN(2n-k) = \left\lfloor \frac{p_{\pi(2n-k)}}{2n-k} \right\rfloor \tag{129}$$

In formulas (128) and (129), $\lfloor\ \rfloor$ is the rounding down operator.

Now multiply the right parts of formulas (127) and (128), summarize and get the number of partitions of an even number into pairs of prime numbers by formula (117):

$$PP(2n) = \sum_{k=2}^{n} \left\lfloor \frac{p_{\pi(k)}}{k} \right\rfloor \cdot \left\lfloor \frac{p_{\pi(2n-k)}}{2n-k} \right\rfloor \tag{130}$$

Let us construct a formula for counting the number of partitions of an even number into pairs of prime numbers using formula (118).

Suppose that there is a sequence of prime numbers p_k not exceeding the even number $2n$.

If $(2n-p_k)$ is a prime number, then $\pi(2n-p_k)$ is the number of that number and we get the analogy of formula (124):

$$p_{\pi(2n-p_k)} = 2n - p_k \tag{131}$$

$$\frac{p_{\pi(2n-p_k)}}{2n-p_k} = 1 \tag{132}$$

The function of checking a number $(2n-p_k)$ for simplicity will look like this:

$$TPN(2n-p_k) = \left\lfloor \frac{p_{\pi(2n-p_k)}}{2n-p_k} \right\rfloor \tag{133}$$

Now let us put expression (133) into the sum function and obtain the function of partitioning an even number into pairs of prime numbers by formula (118).

$$PP(2n) = \sum_{k=1}^{\pi(n)} \left\lfloor \frac{p_{\pi(2n-p_k)}}{2n-p_k} \right\rfloor \tag{134}$$

The formula (134) is correct for $n \geq 2$ and $2n \geq 4$.

Let us construct a formula for calculating the number of partitions of an even number into pairs of prime numbers by formula (119), taking into account formula (133):

$$PP(2n) = \sum_{k=1}^{\lfloor (n-1)/2 \rfloor} \left\lfloor \frac{p_{\pi(2k+1)}}{2k+1} \right\rfloor \cdot \left\lfloor \frac{p_{\pi(2n-2k-1)}}{2n-2k-1} \right\rfloor \tag{135}$$

The formula (135) is correct for $n \geq 3$ and $2n \geq 6$.

Let us construct a formula for calculating the number of partitions of an even number into pairs of prime numbers by formula (120), taking into account formula (133):

$$PP(2n) = \sum_{k=0}^{n-2} \left\lfloor \frac{p_{\pi(n-k)}}{n-k} \right\rfloor \cdot \left\lfloor \frac{p_{\pi(n+k)}}{n+k} \right\rfloor \tag{136}$$

Let us construct a formula for counting the number of partitions of an even number $2n$ into pairs of prime numbers by formula (121), taking into account formula (133), if the number n is odd:

$$PP(2n) = \sum_{k=0}^{(n-3)/2} \left\lfloor \frac{p_{\pi(n-2k)}}{n-2k} \right\rfloor \cdot \left\lfloor \frac{p_{\pi(n+2k)}}{n+2k} \right\rfloor \tag{137}$$

The formula (137) is correct for $n \geq 3$ and $2n \geq 6$.

Let us construct a formula for counting the number of partitions of an even number $2n$ into pairs of prime numbers by formula (122), taking into account formula (133), if the number n is even:

$$PP(2n) = \sum_{k=0}^{(n-4)/2} \left\lfloor \frac{p_{\pi(n-2k-1)}}{n-2k-1} \right\rfloor \cdot \left\lfloor \frac{p_{\pi(n+2k+1)}}{n+2k+1} \right\rfloor \tag{138}$$

The formula (138) is correct for $n \geq 4$ and $2n \geq 8$.

Recall Fermat's small theorem:

$$a^{p-1} \equiv 1 \pmod{p} \tag{139}$$

From this formula, let us take such a component:

$$\frac{a^{p-1} - 1}{p} \tag{140}$$

Let's use the cosine function, just as Willans used it in his formula for the nth prime number.
Let's insert formula (140) into the cosine function.
The functions for checking the numbers k and $(2n-k)$ for simplicity will look like this:

$$TPN(k) = \left\lfloor cos^2 \left(\alpha \cdot \frac{a^{(k-1)} - 1}{k} \right) \right\rfloor \tag{141}$$

$$TPN(2n-k) = \left\lfloor cos^2 \left(\alpha \cdot \frac{a^{(2n-k-1)} - 1}{2n-k} \right) \right\rfloor \tag{142}$$

Here the parameter $\alpha = 180° = const$p is needed to select prime numbers.

If the numbers k and $(2n-k)$ are prime, then $\lfloor cos^2(\alpha \cdot ...) \rfloor = 1$, and if they are composite, then $\lfloor cos^2(\alpha \cdot ...) \rfloor = 0$.

Multiply the right parts of formulas (141) and (142), enter the sign of the sum and get the number of partitions according to formula (117):

$$PP(2n) \ = \ \sum_{k=2}^{n} \left\lfloor cos^2 \left(\alpha \cdot \frac{a^{(k-1)} \ - \ 1}{k} \right) \right\rfloor \cdot \left\lfloor cos^2 \left(\alpha \cdot \frac{a^{(2n-k-1)} \ - \ 1}{2n-k} \right) \right\rfloor \tag{143}$$

If the value of a prime number p_k is known, the function of checking a number $(2n - p_k)$ for simplicity will look like this:

$$TPN(2n-p_k) \ = \ \left\lfloor cos^2 \left(\alpha \cdot \frac{a^{(2n-p_k-1)} \ - \ 1}{2n-p_k} \right) \right\rfloor \tag{144}$$

Put the right side of formula (144) into the sign of the sum and we get a formula for the number of partitions of an even number into pairs of prime numbers by formula (118):

$$PP(2n) \ = \ \sum_{k=1}^{\pi(n)} \left\lfloor cos^2 \left(\alpha \cdot \frac{a^{(2n-p_k-1)} \ - \ 1}{2n-p_k} \right) \right\rfloor \tag{145}$$

Number of partitions of an even number into pairs of prime numbers by formula (119), taking into account formulas (141) and (142):

$$PP(2n) \ = \ \sum_{k=1}^{\lfloor (n-1)/2 \rfloor} \left\lfloor cos^2 \left(\alpha \cdot \frac{a^{(2k)} \ - \ 1}{2k+1} \right) \right\rfloor \cdot \left\lfloor cos^2 \left(\alpha \cdot \frac{a^{(2n-2k-2)} \ - \ 1}{2n-2k-1} \right) \right\rfloor \tag{146}$$

The formula (146) is correct for $n \geq 3$ and $2n \geq 6$.

Let us present a variant of the partition formula without p_k partitioning to formula (120) taking into account formulas (141) and (142):

$$PP(2n) \ = \ \sum_{k=0}^{n-2} \left\lfloor cos^2 \left(\alpha \cdot \frac{a^{(n-k-1)} \ - \ 1}{n-k} \right) \right\rfloor \cdot \left\lfloor cos^2 \left(\alpha \cdot \frac{a^{(n+k-1)} \ - \ 1}{n+k} \right) \right\rfloor \tag{147}$$

Let us construct a formula for counting the number of partitions of an even number $2n$ into pairs of prime numbers by formula (121) if the number n is odd:

$$PP(2n) \ = \ \sum_{k=0}^{(n-3)/2} \left\lfloor cos^2 \left(\alpha \cdot \frac{a^{(n-2k-1)} \ - \ 1}{n-2k} \right) \right\rfloor \cdot \left\lfloor cos^2 \left(\alpha \cdot \frac{a^{(n+2k-1)} \ - \ 1}{n+2k} \right) \right\rfloor \tag{148}$$

The formula (148) is correct for $n \geq 3$ and $2n \geq 6$.

Let us construct a formula for counting the number of partitions of an even number $2n$ into pairs of prime numbers by formula (122) if the number n is even:

$$PP(2n) \ = \ \sum_{k=0}^{(n-4)/2} \left\lfloor cos^2 \left(\alpha \cdot \frac{a^{(n-2k-2)} \ - \ 1}{n-2k-1} \right) \right\rfloor \cdot \left\lfloor cos^2 \left(\alpha \cdot \frac{a^{(n+2k)} \ - \ 1}{n+2k+1} \right) \right\rfloor \tag{149}$$

The formula (149) is correct for $n \geq 4$ and $2n \geq 8$.

Recall Wilson's theorem:

$$(p-1)! \ \equiv \ -1 \pmod{p} \tag{150}$$

From this formula we take the following component:

$$\frac{(p-1)!\ +\ 1}{p} \tag{151}$$

Let us present a variant of partitioning an even number into pairs of prime numbers by formula (117):

$$PP(2n)\ =\ \sum_{k=2}^{n}\left\lfloor cos^2\left(\alpha\cdot\frac{(k-1)!\ +1}{k}\right)\right\rfloor\cdot\left\lfloor cos^2\left(\alpha\cdot\frac{(2n-k-1)!\ +1}{2n-k}\right)\right\rfloor \tag{152}$$

Let's construct a simplicity check function using the cosine function.

The function of checking a number $(2n-p_k)$ for simplicity will look like this:

$$TPN(2n-p_k)\ =\ \left\lfloor cos^2\left(\alpha\cdot\frac{(2n-p_k-1)!\ +1}{2n-p_k}\right)\right\rfloor \tag{153}$$

Put the right side of formula (153) into the sign of the sum and we get a formula for the number of partitions of an even number into pairs of prime numbers by formula (118):

$$PP(2n)\ =\ \sum_{k=1}^{\pi(n)}\left\lfloor cos^2\left(\alpha\cdot\frac{(2n-p_k-1)!\ +1}{2n-p_k}\right)\right\rfloor \tag{154}$$

Now a variant of partitioning an even number into pairs of prime numbers by formula (120):

$$PP(2n)\ =\ \sum_{k=0}^{n-2}\left\lfloor cos^2\left(\alpha\cdot\frac{(n-k-1)!\ +1}{n-k}\right)\right\rfloor\cdot\left\lfloor cos^2\left(\alpha\cdot\frac{(n+k-1)!\ +1}{n+k}\right)\right\rfloor \tag{155}$$

Recall formula $Mináč's$ for counting the number of prime numbers:

$$\pi(n)\ =\ \sum_{j=2}^{n}\left\lfloor\frac{(j-1)!\ +1}{j}\ -\ \left\lfloor\frac{(j-1)!}{j}\right\rfloor\right\rfloor \tag{156}$$

We use the component from formula (156) as a function for checking the simplicity of a number:

$$TPN(j)\ =\ \left\lfloor\frac{(j-1)!\ +1}{j}\ -\ \left\lfloor\frac{(j-1)!}{j}\right\rfloor\right\rfloor \tag{157}$$

In view of the above, the function of partitioning an even number into pairs of prime numbers by formula (117) will be of the form:

$$PP(2n)\ =\ \sum_{k=2}^{n}\left\lfloor\frac{(k-1)!\ +1}{k}\ -\ \left\lfloor\frac{(k-1)!}{k}\right\rfloor\right\rfloor\cdot\left\lfloor\frac{(2n-k-1)!\ +1}{2n-k}\ -\ \left\lfloor\frac{(2n-k-1)!}{2n-k}\right\rfloor\right\rfloor \tag{158}$$

If all values of prime numbers p_k are known in advance, then the partition by formula (118) will look like this:

$$PP(2n)\ =\ \sum_{k=1}^{\pi(n)}\left\lfloor\frac{(2n-p_k-1)!\ +1}{2n-p_k}\ -\ \left\lfloor\frac{(2n-p_k-1)!}{2n-p_k}\right\rfloor\right\rfloor \tag{159}$$

The function of partitioning an even number into pairs of prime numbers by formula (119) will be of the form:

$$PP(2n) \ = \sum_{k=1}^{\lfloor (n-1)/2 \rfloor} \left\lfloor \frac{(2k)! \ +1}{2k+1} - \left\lfloor \frac{(2k)!}{2k+1} \right\rfloor \right\rfloor \cdot \left\lfloor \frac{(2n-2k-2)! \ +1}{2n-2k-1} - \left\lfloor \frac{(2n-2k-2)!}{2n-2k-1} \right\rfloor \right\rfloor \tag{160}$$

The formula (160) is correct for $n \geq 3$ and $2n \geq 6$.

The function of partitioning an even number into pairs of prime numbers by formula (120) will be of the form:

$$PP(2n) = \sum_{k=0}^{n-2} \left\lfloor \frac{(n-k-1)!+1}{n-k} - \left\lfloor \frac{(n-k-1)!}{n-k} \right\rfloor \right\rfloor \cdot \left\lfloor \frac{(n+k-1)!+1}{n+k} - \left\lfloor \frac{(n+k-1)!}{n+k} \right\rfloor \right\rfloor \tag{161}$$

Let us construct a formula for counting the number of partitions of an even number $2n$ into pairs of prime numbers by formula (121) if the number n is odd:

$$PP(2n) = \sum_{k=0}^{(n-3)/2} \left\lfloor \frac{(n-2k-1)!+1}{n-2k} - \left\lfloor \frac{(n-2k-1)!}{n-2k} \right\rfloor \right\rfloor \cdot \left\lfloor \frac{(n+2k-1)!+1}{n+2k} - \left\lfloor \frac{(n+2k-1)!}{n+2k} \right\rfloor \right\rfloor \tag{162}$$

The formula (162) is correct for $n \geq 3$ and $2n \geq 6$.

Let us construct a formula for counting the number of partitions of an even number $2n$ into pairs of prime numbers by formula (122) if the number n is even:

$$PP(2n) = \sum_{k=0}^{(n-4)/2} \left\lfloor \frac{(n-2k-2)!+1}{n-2k-1} - \left\lfloor \frac{(n-2k-2)!}{n-2k-1} \right\rfloor \right\rfloor \cdot \left\lfloor \frac{(n+2k)!+1}{n+2k+1} - \left\lfloor \frac{(n+2k)!}{n+2k+1} \right\rfloor \right\rfloor \tag{163}$$

The formula (163) is correct for $n \geq 4$ and $2n \geq 8$.

As a function for checking the simplicity of a number, we can construct the following formula:

$$TPN(x) \ = \ 1 \ - \ \left\lceil \frac{(x-1)! \ +1}{x} \right\rceil \ + \ \left\lfloor \frac{(x-1)! \ +1}{x} \right\rfloor \tag{164}$$

In the formula (164), $\lceil \ \rceil$ is the rounding up operator, $\lfloor \ \rfloor$ is the rounding down operator.
Then by formula (118) taking into account formula (164) the function of partitioning an even number into pairs of prime numbers will have the following form:

$$PP(2n) \ = \sum_{k=1}^{\pi(n)} \left(1 \ - \ \left\lceil \frac{(2n-p_k-1)! \ +1}{2n-p_k} \right\rceil \ + \ \left\lfloor \frac{(2n-p_k-1)! \ +1}{2n-p_k} \right\rfloor \right) \tag{165}$$

Let's construct another formula for checking the simplicity of a number:

$$TPN(x) \ = \prod_{i=1}^{\pi(\lfloor \sqrt{x} \rfloor)} \left(\left\lceil \frac{x}{p_i} \right\rceil \ - \ \left\lfloor \frac{x}{p_i} \right\rfloor \right) \tag{166}$$

Then by formula (118) taking into account formula (166) the function of partition of an even number into pairs of prime numbers will have the following form:

$$PP(2n) = \sum_{k=1}^{\pi(n)} \left(\prod_{i=1}^{\pi(\lfloor\sqrt{2n-p_k}\rfloor)} \left(\left\lceil \frac{2n-p_k}{p_i} \right\rceil - \left\lfloor \frac{2n-p_k}{p_i} \right\rfloor \right) \right) \tag{167}$$

Another option for checking the simplicity of a number:

$$TPN(x) = \prod_{i=2}^{\lfloor\sqrt{x}\rfloor} \left(\left\lceil \frac{x}{i} \right\rceil - \left\lfloor \frac{x}{i} \right\rfloor \right) \tag{168}$$

Then by formula (117) taking into account formula (168) the function of partitioning an even number into pairs of prime numbers will have the following form:

$$PP(2n) = \sum_{k=2}^{n} \left(\prod_{i=2}^{\lfloor\sqrt{k}\rfloor} \left(\left\lceil \frac{k}{i} \right\rceil - \left\lfloor \frac{k}{i} \right\rfloor \right) \right) \cdot \left(\prod_{j=2}^{\lfloor\sqrt{2n-k}\rfloor} \left(\left\lceil \frac{2n-k}{j} \right\rceil - \left\lfloor \frac{2n-k}{j} \right\rfloor \right) \right) \tag{169}$$

According to formula (119), taking into account formula (168), the function of partitioning an even number into pairs of prime numbers will have the following form:

$$PP(2n) = \sum_{k=2}^{n} \left(\prod_{i=2}^{\lfloor\sqrt{2k+1}\rfloor} \left(\left\lceil \frac{2k+1}{i} \right\rceil - \left\lfloor \frac{2k+1}{i} \right\rfloor \right) \right) \cdot \left(\prod_{j=2}^{\lfloor\sqrt{2n-2k-1}\rfloor} \left(\left\lceil \frac{2n-2k-1}{j} \right\rceil - \left\lfloor \frac{2n-2k-1}{j} \right\rfloor \right) \right) \tag{170}$$

By formula (120) the function of partitioning an even number into pairs of prime numbers will have the following form:

$$PP(2n) = \sum_{k=0}^{n-2} \left(\prod_{i=2}^{\lfloor\sqrt{n-k}\rfloor} \left(\left\lceil \frac{n-k}{i} \right\rceil - \left\lfloor \frac{n-k}{i} \right\rfloor \right) \right) \cdot \left(\prod_{j=2}^{\lfloor\sqrt{n+k}\rfloor} \left(\left\lceil \frac{n+k}{j} \right\rceil - \left\lfloor \frac{n+k}{j} \right\rfloor \right) \right) \tag{171}$$

By formula (121) the function of partitioning an even number into pairs of prime numbers will have the following form:

$$PP(2n) = \sum_{k=0}^{(n-3)/2} \left(\prod_{i=2}^{\lfloor\sqrt{n-2k}\rfloor} \left(\left\lceil \frac{n-2k}{i} \right\rceil - \left\lfloor \frac{n-2k}{i} \right\rfloor \right) \right) \cdot \left(\prod_{j=2}^{\lfloor\sqrt{n+2k}\rfloor} \left(\left\lceil \frac{n+2k}{j} \right\rceil - \left\lfloor \frac{n+2k}{j} \right\rfloor \right) \right) \tag{172}$$

By formula (122) the function of partitioning an even number into pairs of prime numbers will have the following form:

$$PP(2n) = \sum_{k=0}^{(n-4)/2} \left(\prod_{i=2}^{\lfloor\sqrt{n-2k-1}\rfloor} \left(\left\lceil \frac{n-2k-1}{i} \right\rceil - \left\lfloor \frac{n-2k-1}{i} \right\rfloor \right) \right) \times$$
$$\times \left(\prod_{j=2}^{\lfloor\sqrt{n+2k+1}\rfloor} \left(\left\lceil \frac{n+2k+1}{j} \right\rceil - \left\lfloor \frac{n+2k+1}{j} \right\rfloor \right) \right) \tag{173}$$

Conclusions.

Exact and approximate formulas for calculating the number of partitions of even numbers into pairs of prime numbers were obtained.

Specific exact values of the numbers of partitions of several even numbers into pairs of prime numbers were also computed and presented in the form of a table.

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